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THE ADVANTAGES OF PRODUCTION TESTING AT VARIABLE RATES

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ABSTRACT

Flow rates occurring during production and interference tests are seldom constant. This paper presents an analytical method of solving for pressure as a function of time for variable flow rates. The cases examined are for linear, exponential and harmonic production declines.

The analysis of a variable flow rate can provide more information than the analysis of a constant rate. The procedure is simple and does not require the the application of superposition. To force a well to produce at a constant rate or to assume that it produced in a step like manner may complicate the arithmetic, induce errors and often restrict the number of parameters that can be solved.

The variable rate technique is particularly useful where the flow rate is continuously monitored. A common case is a drill stem test where the production can be determined accurately as the fluid level rises in the string and the rate is continuously changing with time. In the case where a well dies during a test this technique has proved very useful.

INTRODUCTION

In conducting production and interference tests it has been customary to attempt to maintain the rate at constant conditions. Clearly the reason is related to the solution of the radial flow equation which for constant rate is well known. In the case where this condition is not met superposition or a Horner[1] type correction is applied. These are approximations which may not be necessary. The advent of continuous data monitoring and the ease with which arithmetic can be manipulated by computer makes it possible to investigate other solutions.

The classical build-up analysis where production is assumed to cease instantaneously is a mathematical simplification. After shutin the reservoir continues to produce and for this reason it has been necessary to introduce such complexities as after-flow and well-bore storage. In some cases it may be preferable to analyse such a test using a harmonic production decline. To open or shut in a well suddenly may be mechanically inadvisable and mathematically unnecessary.

Experience has shown that it is preferable to allow a well to respond naturally rather than to force a constant rate.

- The mechanical operation is simplified.
- The flow period can be reduced because it is not necessary for the rate to stabilize.
- In some cases it is possible to solve for more reservoir parameters and the solution may be more accurate.

THEORY

The techniques used to interpret production tests are generally based on the analytical solution of the continuity equation which is a combination of Darcy's law and fluid storage. The subject has been addressed in the literature in detail for many reservoir conditions such as heterogeneity and anisotropy[2]. In most cases however, the assumption is made that the flow rate is constant over the entire test or over discrete periods which make up the test. The validity of such assumptions is seldom questioned. The resulting errors can be significant. This was noted when the equations presented in this paper were tested against the results derived from a simulator and from superposition techniques. To generate comparably accurate results by these techniques the flow period had to be divided into a very large number of time steps.

The equations presented in this paper assume radial flow conditions for a single phase in an infinitely acting, homogenous reservoir. For a constant rate the solution for pressure as a function of time and space was given by Theis[3].

The Constant Rate equation

$$DP = \frac{70.6 \cdot q \cdot \mu \cdot \beta}{k \cdot h} \cdot \left[\int_{\lambda}^{\infty} \frac{e^{-\lambda}}{\lambda}\, d\lambda + 2 \cdot s \right]$$

$$DP = m \cdot q \cdot \left[E_i(\lambda) + 2 \cdot s \right] \quad \text{where}$$

$$q \text{ is constant and } \lambda := \frac{\phi \cdot \mu \cdot c \cdot r^2}{.00105 \cdot k \cdot t}$$

Stallman[4] showed that by superposition this equation can be used to analyze variable rates. This method however, requires a considerable amount of arithmetic if the flow rate changes rapidly and continuously.

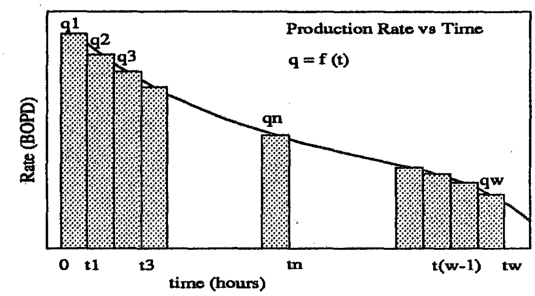
The Superposition Equation

$$DP_{tw} = m \cdot \left[q_1 \cdot \left[\int_{\lambda(tw)}^{\infty} \frac{e^{-\lambda}}{\lambda}\, d\lambda - \int_{\lambda(tw - t1)}^{\infty} \frac{e^{-\lambda}}{\lambda}\, d\lambda \right] + \right.$$

$$+ q_2 \cdot \left[\int_{\lambda(tw - t1)}^{\infty} \frac{e^{-\lambda}}{\lambda}\, d\lambda - \int_{\lambda(tw - t2)}^{\infty} \frac{e^{-\lambda}}{\lambda}\, d\lambda \right] + q_3 \cdot \ldots\ldots$$

$$\left. + q_w \cdot \left[\int_{\lambda(tw - t(w - 1))}^{\infty} \frac{e^{\lambda}}{\lambda}\, d\lambda + 2 \cdot s \right] \right]$$

It is possible to solve for the pressure change DP for variable flow rate conditions by using the transform $T = (t_w - t)$. Both the pressure change DP and the flow rate q then become functions of T. The transform describes the time between the beginning of the test and t_w which is the time when DP is evaluated. This is visualized as follows:



From the superposition equation as shown above

$$DP_{tw} = m \cdot \left[q_1 \cdot \int_{\lambda(tw)}^{\lambda(tw - t1)} \frac{e^{-\lambda}}{\lambda}\, d\lambda + q_2 \int_{\lambda(tw - t1)}^{\lambda(tw - t2)} \frac{e^{-\lambda}}{\lambda}\, d\lambda + \right.$$

$$+ q_n \cdot \int_{\lambda(tw - tn)}^{\lambda(tw - t(n - 1))} \frac{e^{-\lambda}}{\lambda}\, d\lambda + \ldots\ldots\ldots\ldots$$

$$\left. \ldots\ldots\ldots\ldots \quad + q_w \left[\int_{\lambda tw}^{\infty} \frac{e^{-\lambda}}{\lambda}\, d\lambda + 2 \cdot S \right] \right]$$

in the limit as (tw-tn)/(tw - t(n-1)) approaches 1 and

T = (tw - t) and q(w) is the rate at tw then

$$DP_{tw} = m \cdot \left[\int_{\lambda}^{\infty} q(T) \cdot \frac{e^{-\lambda}}{\lambda}\, d\lambda + 2 \cdot S \cdot q(w) \right] \quad \ldots\ldots\ \text{Equation 1}$$

which is ...

The Variable Flow Rate Equation

For those who would like a more rigorous solution reference is made to Carslow & Jager[5] who describe the line source equation in detail. The above is a very important equation which has not been greatly used in reservoir engineering.

This general equation can be solved for any rate q as a monotonic function of time t. The mathematics and the resulting arithmetic are often simple and pleasing. The cases presented in this paper assume the following flow rate functions:

- Constant change of rate.
- Exponential change of rate.
- Harmonic change of rate.

The derivations of the solutions to equation 1 for each of the above conditions are given in appendices A, B & C. These flow rate functions have sufficient parameters to fit most curves. The many graphical and mathematical computer packages available today make it possible to match recorded flow rates with one of these functions.

THE SOLUTIONS

Presented below is the solution for each of the above flow rate functions. This is the change in reservoir pressure (DP) from initial condition P_i at time zero to P_{tw} at time t_w.

For each solution three cases are presented:

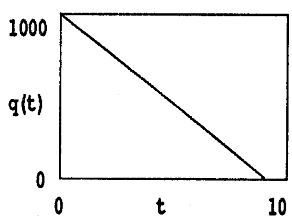
- At any time and at any distance.
- At any time at rw.
- At rw when the well stops flowing.

The first solution applies to either an observation well or at very early times to the producing well. The second and third solutions clearly apply to the producing well. It should also be noted that the first two solutions are valid for either increasing or decreasing production rates. The last solution refers to a declining rate which can be extrapolated to zero.

THE CONSTANT CHANGE OF RATE

If the decline in production rate is linear with time the solution is relatively simple because the T transform is not necessary.

If the production rate is linear with time



$$q_t = q_i\cdot(1 - B\cdot t)$$

Note B is not dependent on t

At any values of r and t

$$DP_t = m\cdot q_i\cdot\left[\left[(1 - B\cdot t)\cdot\left[E_i(\lambda) + 2\cdot S\right] + B\cdot b\cdot\left[\frac{e^{-\lambda}}{\lambda} - Ei(\lambda)\right]\right]\right]$$

At r = rw where λ is typically small

$$DP_t = m\cdot q_i\cdot((1 - B\cdot t)\cdot(2\cdot S - .5772 - \ln(\lambda)) + B\cdot t)$$

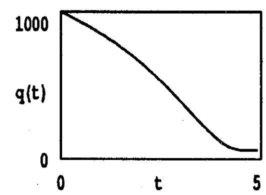
When the well stops flowing

$$DP_t = m\cdot q_i$$

THE EXPONENTIAL CHANGE OF RATE

These equations can be used to match a relatively complex curve. In less complex cases some of the parameters can be dropped by equating them to zero or unity.

If the production rate is exponential with respect to time



$0 \le t \le tw$

$$q(t) := q_i\cdot\left[\lambda + B\cdot e^{-\left[\frac{C}{tw-t}\right]}\right]$$

Note A B & C are dependent on tw

At any values of r and tw

$$DP_{tw} = m\cdot q_i\cdot\left[\lambda\cdot\left[2\cdot S + E_i(\lambda)\right] + B\cdot E_i\left[\lambda\cdot\left[1 + \frac{C}{b}\right]\right]\right]$$

At r = rw where λ is typically small

$$DP_{tw} = m\cdot q_i\cdot\left[\lambda\cdot(2\cdot S - .5772 - \ln(\lambda)) + B\cdot E_i\left[\frac{C}{tw}\right]\right]$$

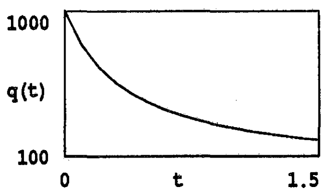
When the well stops flowing

$$DP_{tw} = m\cdot q\cdot\left[(-B)\cdot E_i\left[\frac{C}{tw}\right]\right]$$

THE HARMONIC CHANGE OF RATE

These equations are particularly useful in the case where the well dies gradually.

If the rate is harmonic with respect to time



$0 \le t \le tw$

$$q = \frac{q_i}{A - B\cdot(tw - t)}$$

Note A & B are dependent on tw

At any values of r and tw

$$DP_{tw} = m\cdot q_i \cdot\left[\frac{e^{\frac{\lambda}{A}-\lambda}}{A}\cdot E_i\left[\frac{\lambda}{A}\right] + 2\cdot\frac{S}{A}\right]$$

When r = rw where λ is typically small

$$DP_{tw} = m\cdot q_i \cdot\frac{1}{A}\cdot\left[2\cdot S - .5772 - \ln\left[\frac{\lambda}{A}\right]\right]$$

When the well stops flowing

$$DP_{tw} = m\cdot q_i \cdot\left[\frac{1}{A}\cdot\ln(A)\right]$$

APPLICATION

The following procedure can be used in a wide range of practical problems where the production rate changes monotonically with time.

- Determine which of the following equations best describes the change in flow rate.
  $q = q_i\,(1 + B\,t)$
  $q = q_i\,(1 + B\,e^{-C/t})$
  $q = q_i\,(1/(A + B\,t))$
  Use regression analysis or graphical techniques to establish the values for A B and C. In most cases one or two parameters may be sufficient; the others may be set to zero or unity.

- If the function is not linear transform the flow rate function into:

$q = q_i\,(1 + B\,e^{-C/(t_w - t)})$
$q = q_i\,(1/(A + B\,(t_w - t)))$

- Arrange as many DP(t) equations as the required numbers of parameters to be solved. This can be done sequentially, simultaneously or graphically. If a zero flow rate can be extrapolated from the decline, then permeability is solved by one of the following equations which are linear with respect to k.

WHEN THE FLOW RATE CAN BE EXTRAPOLATED TO ZERO

FLOW RATE FUNCTION	EQUATION
Linear	$k = DP\,h\,/\,(70.6\,q_i\,\mu\,\beta)$
Exponential	$k = DP\,h/(70.6\,q_i\,\mu\,\beta)\,(-b)Ei(C/tw)$
Harmonic	$k = DP\,h\,/\,(70.6\,q_i\,\mu\,\beta)\,(\ln(A)/A)$

This should be done as a first approximation in any case.

- Test the solution for the entire flow period.

THE CASE OF THE DYING WELL

This is one of the many cases where the above technique proved useful. A drill stem test was undertaken in an exploration well.

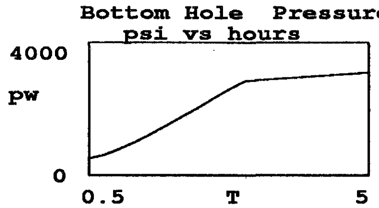
The bottom valve was opened with about 1000 ft of water cushion. The excitement on the rig floor reflected the bubble blow in the bucket. Within 3 hours the well had died and incredulity had set in.

Seismic indicated a large gentle structure with four way closure. This was a proven oil region. The logs showed a clean sand, 32 ft in thickness saturated with oil at a depth of 7000 ft. A twelve ft core confirmed the logs and indicated permeability of 300 md. It was difficult to believe that this well would not produce clean oil at high rates.

Mechanical problems were suspected. Work continued for several days to make the well flow to the surface. The test was subsequently repeated by utilizing a pump. All this was

unnecessary however, because the pressure recorded at bottom during the first three hours told the entire story.
From the bottom hole pressure measurements it was possible to determine accurately the flow rate, the permeability, the skin and also the viscosity of the crude. It was heavy oil with little gas. This analysis was used to reconcile permeability viscosity and compressibility. These values were subsequently cofirmed by laboratory analysis of the crude.

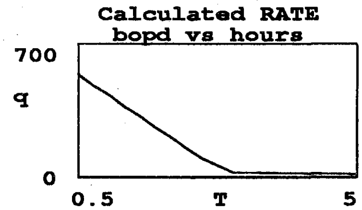
The production rate was calculated from the bottom hole pressure



$DP_{2.9} = 355$

$DP_{2.0} = 1430$

$DP_{0.70} = 2600$



The rate function was thus estimated as follows

$q_i := 650$ and $B := 0.34$ in $q(t) := q_i \cdot (1 - B \cdot t)$

In accordance with the above decline the well stopped flowing at 2.94 hours

At zero rate $DP = 70.6\ q\ \mu\ \beta/(k\ h) = m\,q_i$ psi

Calculating $k/\mu = (70.6\ 650\ 1.04/(32\ 355) = 4.2$ md/cp

Then $\lambda = \phi\ \mu\ c\ r\ r/(.00105\ k\ t) = 0.2\ (1/4.2)\ 12E\text{-}6\ .4\ .4\ /(.00105\ t)$
$= .0000871/t$

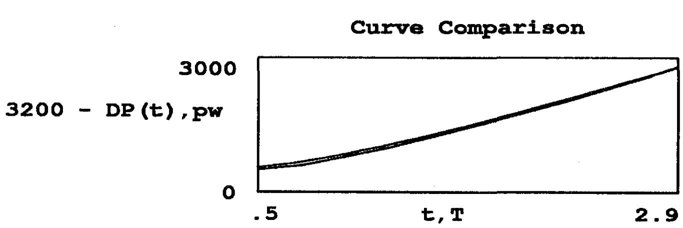
But $DP = m\ qi\ ((1 - Bt)(2S - .5772 - \ln(\lambda) + Bt)$ psi

Therefore $S = 0.5((DP/355 - .34t)/(1-.34t) + .577 + \ln(.0000871/t))$

Calculating Skin @ t=2 S = 0.44 as a check @t = 0.7 S = 0.49

Check the above values for the entire curve for $0.5 \le t \le 2.9$ hrs

by substitution in $DP(t) = 355((1-Bt)\ (2\ 0.5 - .5772 - \ln(.0000871/t)) + Bt)$



Where
3200-DP(t) was calculated
and
pw was measured

It was thus concluded that k = 300 md

and S = 0.5 and $\mu = 300 / 4.2 = 72$ cp

CONCLUSION

Where a production or interference test is carried out at a rate which varies monotonically with time it is not necessary to assume a constant rate or to transform the production profile into a step function. Analytical solutions which utilize variable flow rates are accurate and easy to use.

To force a well to produce at a constant rate or to assume constant rate conditions where they do not exist may be unjustifiable. If the flow can be matched by a monotonic function, the pressure in the reservoir, can be calculated by relatively simple variable flow rate equations. These equations can be used to solve for several reservoir parameters such as permeability and skin. In some cases the arithmetic required by this technique is considerably simpler than that of superposition.

NOMENCLATURE

A B C	= Parameters in q = qi f(t)
c	= Compressibility 1/psi
b = ϕ μ C r r /(.00105 k)	= λ t hours
Ei (λ)	= $\int_{\lambda}^{\infty} \frac{e^{-\lambda}}{\lambda} d\lambda$
h	= net pay ft
i	= initial conditions (subscript)
w	= final conditions (subscript)
k	= Permeability md
m	= 70.6 μ β / (k h) psi/bpd
P	= Pressure psi
q	= Flow rate bpd
r	= Radius ft
S	= Skin
t	= time hours
T = (tw - t)	= transform hours
β	= FVF bbl / STB
μ	= Viscosity cp
λ	= ϕ μ C r r /(.00105 k t)
ϕ	= Porosity
ρ	= Transform
DP	= Pressure drop (Pi - Pt)
f	= Function

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APPENDIX A

THE LINEAR FLOW RATE FUNCTION

To determine DP if $q := q_i \cdot (1 - B \cdot t)$

if $T := tw - t$ $\quad q := q_i \cdot (1 - B \cdot (tw - T))$ and if $m = 70.6 \cdot \mu \cdot \frac{\beta}{k \cdot h}$

From Equation 1 ...... $$DP = m \cdot q_i \cdot \left[\int_{\lambda}^{\infty} ((1 - Btw) + BT) \cdot \frac{e^{-\lambda}}{\lambda} \, d\lambda + (1 - Bt) \cdot 2 \cdot S \right]$$

$$= m \cdot q_i \cdot \left[(1 - B \cdot tw) \cdot \int_{\lambda}^{\infty} e^{-\lambda} \, d\lambda + B \cdot \int_{\lambda}^{\infty} T \cdot \frac{e^{-\lambda}}{\lambda} \, d\lambda + (1 - B \cdot t) \cdot 2 \cdot S \right]$$

Considering the 2nd integral in the above equation 1 A

because $\lambda = \frac{b}{T}$ then $$\frac{1}{b} \cdot \int_{\lambda}^{\infty} T \cdot \frac{e^{-\lambda}}{\lambda} \, d\lambda = \int_{\lambda}^{\infty} \frac{e^{-\lambda}}{\lambda \cdot \lambda} \, d\lambda$$

$$= \left. \frac{-1}{\lambda} - \ln(\lambda) + \frac{\lambda}{2!} - \frac{\lambda^2}{2 \cdot 3!} + \frac{\lambda^3}{3 \cdot 4!} - \ldots\ldots + \frac{\lambda^n}{n \cdot (n + 1)!} - \ldots \right|_{\lambda}^{\infty}$$

$$= \left. \frac{-1}{\lambda} - \ln(\lambda) + \left[\lambda - \frac{\lambda}{2!} \right] - \left[\frac{\lambda^2}{2.2!} - \frac{\lambda^2}{3!} \right] + \left[\frac{\lambda^3}{3.3!} - \frac{\lambda^3}{4!} \right] - \ldots \right|_{\lambda}^{\infty}$$

$$= \left. -1 - \frac{1}{\lambda} \left[1 - \lambda + \frac{\lambda^2}{2!} - \frac{\lambda^3}{3!} + \ldots \right] - \left[\ln(\lambda) - \lambda + \frac{\lambda^2}{2 \cdot 2!} - \frac{\lambda^3}{3.3!} + \ldots \right] \right|_{\lambda}^{\infty}$$

$$= -E_i(\lambda) - \left[1 + \frac{1}{\lambda} \cdot e^{-\lambda} \right]_{\lambda}^{\infty}$$ which is substituted in equation 1A

For any value of t and r $$DP = m \cdot q_i \left[(1 - Bt) E_i(\lambda) + Bb \left[\frac{e^{-\lambda}}{\lambda} - E_i(\lambda) \right] + 2 \cdot (1 - Bt) \cdot S \right]$$

at $r = rw$ λ is small $$DP = m \cdot q_i ((1 - Bt) \cdot (2 \cdot S - .5772 - \ln(\lambda)) + Bt)$$

When the well stops producing $Bt = 1$ then $DP = m \cdot q_i$ ie $k = 70.6 \cdot q_i \cdot \mu \cdot \frac{\beta}{h \cdot DP}$

APPENDIX B

THE EXPONENTIAL RATE FUNCTION

To determine DP at tw if $$q = q_i \cdot \left[A + B \cdot e^{-\left[\frac{C}{tw-t}\right]}\right]$$

If $T = tw - t$ and $\lambda = b/T$ $$q = q_i \cdot \left[A + B \cdot e^{-\left[\frac{C}{b} \cdot \lambda\right]}\right]$$

From equation 1

$$DP = m \cdot q_i \cdot \left[A \cdot \int_{\lambda}^{\infty} \frac{e^{-\lambda}}{\lambda} d\lambda + B \cdot \int_{\lambda}^{\infty} \frac{e^{-\left[\lambda \cdot \left[1+\frac{C}{b}\right]\right]}}{\lambda} d\lambda + 2 \cdot S \cdot A\right]$$

but if $\rho = \lambda\ (1 + c/b)$ $$\int_{\lambda}^{\infty} e^{-\left[\lambda \cdot \left[1+\frac{c}{b}\right]\right]} d\lambda := \int_{\rho}^{\infty} e^{-\rho} d\rho = Ei(\rho)$$

The general solution for any value of r and tw is

$$DP = m \cdot q_i \cdot \left[A \cdot \left[2 \cdot S + E_i(\lambda)\right] + B \cdot E_i \left[\lambda \cdot \left[1 + \frac{C}{b}\right]\right]\right]$$

In the case of the producing well where r = rw and λ is therefore small

$Ei \approx -.5772 - \ln(\lambda)$ and $\lambda \cdot \left[1 + \frac{C}{b}\right] \approx \frac{C}{t}$

$$DP = m \cdot q_i \cdot \left[A \cdot (2 \cdot S - .5772 - \ln(\lambda)) + B \cdot \left[E_i \left[\frac{C}{t}\right]\right]\right]$$

In the case where the well stops producing at tw then A =0

$$DP = m \cdot q_i \cdot \left[B \cdot E_i \left[\frac{C}{t}\right]\right] \quad \text{ie} \quad k = 70.6 \cdot q_i \cdot \mu \cdot \frac{\beta}{DP \cdot h} \cdot \left[-B \cdot E_i \left[\frac{C}{t}\right]\right]$$

APPENDIX C

THE HARMONIC FLOW RATE FUNCTION

To determine DP at tw if $q := \dfrac{q_i}{A + B\cdot(tw - t)}$ If $T = tw - t$ and $\lambda = b / T$

then $q = q_i \cdot \dfrac{\lambda}{A\cdot\lambda + b\cdot B}$ From equation 1

$$DP = m\cdot q_i \cdot \left[\int_{\lambda}^{\infty} \frac{e^{-\lambda}}{A\cdot\lambda + b\cdot B}\, d\lambda + 2\cdot\frac{S}{A}\right]$$

but if $\rho = \lambda + b\,B / A$

$$\int_{\lambda}^{\infty} \frac{e^{-\lambda}}{A\cdot\lambda + B\cdot b}\, d\lambda := e^{-\left[b\cdot\frac{B}{A}\right]} \cdot \int_{\rho}^{\infty} \frac{e^{-\rho}}{\rho}\, d\rho = \frac{e^{-\left[b\cdot\frac{B}{A}\right]}}{A}\cdot E_i\left[\lambda + b\cdot\frac{B}{A}\right]$$

Because $B = 1/t - A/tw$ therefore $\lambda + b\,B/A = \lambda / A$

For any value of r and t $DP = m\cdot q_i \cdot \left[\dfrac{e^{\frac{\lambda}{A}-\lambda}}{A}\cdot E_i\left[\dfrac{\lambda}{A}\right] + 2\cdot\dfrac{S}{A}\right]$

In the producer $r = rw$ and λ is small $DP = m\cdot q_i \cdot \dfrac{1}{A}\cdot\left[-.5772 - \ln\left[\dfrac{\lambda}{A}\right] + 2\cdot S\right]$

If the well stops flowing it is necessary to introduce another term as follows

$q = q_i \cdot \left[\dfrac{1}{a + B\cdot(tw - t)} - C\right]$ then $DP = m\cdot q_i \left[\dfrac{1}{A}\cdot\left[-.5772 - \ln\left[\dfrac{\lambda}{A}\right]\right] - C\cdot(-.5772 - \ln(\lambda))\right]$

but if the well stops producing at tw then $1/A = C$ then

$DP = m\cdot q_i \cdot \left[\dfrac{1}{A}\cdot\ln(A)\right]$ ie $k = 70.6\cdot q_i \cdot\mu\cdot \dfrac{\beta}{DP\cdot h}\cdot\left[\dfrac{1}{A}\cdot\ln(A)\right]$